

Superconducting $d_{x^2-y^2} \pm id_{xy}$ phase glass

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We discuss the effects of magnetic impurities on d-wave superconductors. We calculate the electron mediated RKKY interaction between the impurity spins in a d-wave superconductor and find that it decays as r^{-3} at large distances. We argue that this interaction leads to the formation of a spin glass at low temperature $T \ll T_c$. It was previously shown that a local complex $\Delta^1 \sim d_{xy}$ order parameter is induced around each impurity spin. We consider the pair tunneling resulting in the Josephson interaction between different patches of induced d_{xy} order parameter. Due to the local coupling between impurity spins and the superconducting order parameter the Josephson coupling favors a ferromagnetic phase at low temperatures. The competition between the Josephson coupling and the RKKY interaction gives rise to an interesting phase diagram. At low impurity concentrations we find an unusual superconducting phase glass, where the impurity spins S^z and d_{xy} component are disordered and yet the product of these two develops a true long range order $\langle S^z \Delta^1 \rangle$. This phase has no analog in purely magnetic spin glasses and arises as a result of the direct coupling of the impurity spin to the phase of Δ^1 . At high impurity concentrations it is possible that a ferromagnetic phase will form.

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I. INTRODUCTION

It has been known for a long time that magnetic ions in a superconductor can form a spin glass (SG) state.¹ There is growing experimental evidence for the formation of a SG phase in the high- T_c materials once magnetic ions, such as Fe and Ni, are doped into the system.² Since magnetic ions interact with the superconducting condensate it is natural to expect³ that this interaction will cause frustration of the underlying condensate and eventually might lead to a superconducting glass or phase glass (PG).

In all of the above discussions of the role of impurity spins it was assumed that magnetic scattering frustrates and suppresses the superconductivity. There are additional physical effects that were not addressed in previous work: namely how frustrated localized spins in an unconventional superconductors can *distort* the condensate and produce *patches* of secondary components of the order parameter near the localized impurity, see Fig. 1. It has been shown that a magnetic impurity in a $d_{x^2-y^2}$ superconductor with order parameter Δ^0 induces a local complex d_{xy} component of the order parameter which we designate Δ^1 .⁴ Locally near each impurity site, on the scale of the coherence length ξ , there is a patch of $\Delta^0 + i\Delta^1$ order parameter for $S^z = +1$ and $\Delta^0 - i\Delta^1$ for $S^z = -1$, depending on the sign of the impurity spin. Hereafter we assume classical spins with $S = 1$. This is a reasonable assumption taken the fact that magnetic ions substituted into high- T_c superconductors have a large spins, such as Ni ($S=1$).

Here we will explore the coupling between different patches due to the Josephson coupling. Phase coherence between these patches of $\Delta^0 \pm i\Delta^1$ would lower the ki-

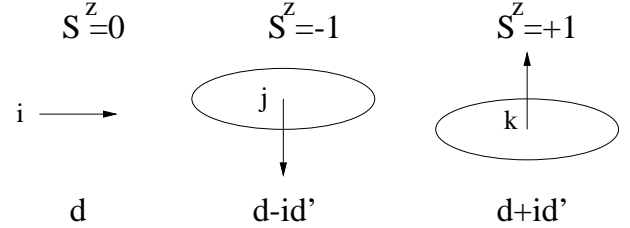


FIG. 1. Three impurity spins at sites i , j and k in a two-dimensional $d_{x^2-y^2}$ superconductor. The patches around site j and k indicate the induced $d' = d_{xy}$ component of the order parameter. Note that there is no induced order parameter around spin i , since the spin is pointing in the xy plane.

netic energy of the condensate and would tend to align all the patches into *globally coherent* $\Delta^0 + i\Delta^1$, or its conjugate. This would imply ferromagnetic ordering of the impurity spins S^z . On the other hand, the dipolar and RKKY spin-spin interaction terms, which we show are mainly antiferromagnetic, would frustrate this ferromagnetic order. In fact, we find that the spin-spin interaction is frustrating and produces a SG phase at $T < T_{SG}$ in the absence of a coupling between the superconducting condensate and spin degrees of freedom. When this coupling is considered the result depends on the relative strength of the two interactions. We find that since the spin-dependent part of the Josephson coupling is small the magnetic subsystem will drive the phase transition, at least at low and intermediate impurity concentrations. Given that the impurity spins form a SG phase we are led to the question how the phase of the induced component $\pm i\Delta^1$ is affected by spin frustration. We find that the spin frustration in the SG phase will frustrate the phase of the d_{xy} component and a PG will form at low tempera-

tures $T < T_{PG} < T_{SG}$. More generally we will discuss the possible phases of the coupled spin-phase model, and at higher impurity concentrations we will consider the formation of a ferromagnetic phase. When discussing glassy phases, we will consider states where the spatial average $\langle S^z \rangle = 0$. However at any given patch, i , the time averaged $\langle S_i^z \rangle_\tau \neq 0$. Similarly we will consider site and time averaged induced component $\langle \Delta^1 \rangle = 0$ and $\langle \Delta_i^1 \rangle_\tau \neq 0$ respectively.

The additional new order parameter we find relevant is the product $\langle S^z \Delta^1 \rangle$. The new order in this case arises from the possibility of having fully disordered phases $\langle \Delta^1 \rangle = \langle S^z \rangle = 0$ and still having the *true long range order* in $\langle S^z \Delta^z \rangle \neq 0$. Physically it corresponds to phase locking of the patches to the local value of impurity spin S^z . This order parameter has no analog in purely magnetic spin glass systems and is a direct consequence of the spin-phase order parameter coupling discussed above. In all of the discussion hereafter we assume that the “backbone” order parameter Δ^0 has true long range order as it is robust at low temperatures $T \ll T_c \sim 100K$.

The plan of this paper is as follows: in the first section we explore the effective spin-spin coupling and integrate out the superconducting degrees of freedom by performing an RKKY calculation. The effective spin model is found to give rise to a SG phase. In the second section of the paper the Josephson coupling between different patches is considered. It is found that the interaction favors a FM phase. In the last section we discuss the PG phase, where the induced superconducting order parameter is locked to the impurity spin. Furthermore we will discuss the possibilities of para- and ferromagnetic phases.

In related recent work⁵ Simon and Varma treat the magnetic impurity problem by a variational approach. They concentrate on the single impurity, but conclude by arguing that the formation of a ferromagnetic phase is unlikely and that a spin glass state is much more likely.

II. EFFECTIVE SPIN MODEL

In this section we will consider the effective spin model that describes the interaction between impurity spins surrounded by patches of induced complex d_{xy} order parameter, as shown in Fig. 1. The most relevant sources of interaction between the impurity spins are the electron-mediated RKKY-interaction and the direct dipolar magnetic interaction. The standard RKKY interaction⁶ is mediated through an interaction of the form $JS \cdot I$, where S refers to the spin of the conduction electron, and I denotes the spin of the impurity. Of particular interest to this work, however, is another term that is directly related to the induced order parameter. The $L \cdot I$ interaction, where L refers to the angular momentum of a conduction electron, scatters an electron into the complex d_{xy} phase. A second order RKKY calculation where

the interaction potential is taken to be of the form $L \cdot I$ will therefore be relevant in this context. The effective spin Hamiltonian is thus of the form

$$H = H^M + H^{S \cdot I} + H^{L \cdot I}, \quad (1)$$

where the dipolar term H^M and the electron mediated interactions $H^{S \cdot I}$ and $H^{L \cdot I}$ are given by

$$H^M = \sum_{ij} \frac{1}{r_{ij}^3} [S_i \cdot S_j - 3(S_i \cdot \hat{r}_{ij})(S_j \cdot \hat{r}_{ij})] \quad (2)$$

$$H^{S \cdot I} = \sum_{ij} J_{ij}^{S \cdot I} S_i \cdot S_j \quad (3)$$

$$H^{L \cdot I} = \sum_{ij} J_{ij}^{L \cdot I} S_i^z S_j^z \quad (4)$$

The size of the dipolar term for two spins separated by 1 Å can be estimated to be about 10^{-4} eV, while the RKKY interaction depends strongly on the coupling between the conduction electrons and the impurity spin. For Mn ions in alloys it has been estimated to be on the order of 10^{-1} eV.⁷ We are not aware of direct estimates of the RKKY interaction in high- T_c materials, but it is likely to be greater than the dipolar interaction. The spatial decay of the last two terms are explicitly calculated in the appendix and are of the form

$$H^{L \cdot I} \sim \frac{2J^2 I_1^z I_2^z k_F^2}{v r^3} [0.37 + 0.33 \sin(2Kr)] \quad (5)$$

$$H^{S \cdot I} \sim \frac{J^2 I_1 \cdot I_2}{2\pi v r^3} [0.37 + 0.33 \sin(2Kr)] \quad (6)$$

where v is the gap velocity and K is a momentum cut-off. The RKKY terms thus share a cubic decay with the dipolar interaction, but are found to be completely antiferromagnetic. However, independently of the specific form of the terms, general consideration tells us that as long as there are sufficient amounts of disorder and frustration present the SG phase should be realized.⁸ The impurity spins we are considering are randomly distributed, and antiferromagnetic interactions will therefore lead to frustration. So as long as the effective interactions are not primarily ferromagnetic the model should have a SG groundstate. The dipolar interaction is antiferromagnetic in the z -component of the spins in an xy plane, and ferromagnetic in the plane. As mentioned above the RKKY terms are completely antiferromagnetic. We therefore conclude that the effective spin model, consisting of the combined dipolar and RKKY terms, exhibits a spin glass phase at low temperatures. For large enough quantum fluctuations it would also be possible to realize a quantum paramagnet phase, where any freezing is destroyed by quantum fluctuations. We focus here on large classical spins and assume that quantum fluctuations are negligible. In principle similar considerations can be given for $S = 1/2$ impurity spins, in which case the paramagnetic phase could be realized.

Note in particular that the impurity spins would form a spin glass even without the $H^{L\cdot I}$ term, and that this term is second order in $\Delta^1 \ll \Delta^0$. This is the motivation why we have first considered the effective spin model and determined the spin configuration. So although the spin and superconducting degrees of freedom are connected, as we will see in the next section, we will there take the spin configuration as given, and determine the phase of the superconducting order parameter based on this. Note also that the importance of the dipolar term may increase as the Δ_1 component is induced, since this will open up a fully gapped state which will tend to exponentially decrease the long-range RKKY interaction. The short-range RKKY interaction should not be significantly affected by the opening of the superconducting gap.

III. JOSEPHSON COUPLING

In this section we will consider the Josephson coupling between the different patches of induced order parameter. Let us first consider the order parameter around an impurity spin

$$\Psi_i = (\Delta_i^0 + e^{i\frac{\pi}{2}S_i^z}\Delta_i^1)e^{i\theta_i}, \quad (7)$$

where the real parameter Δ^0 denotes the $d_{x^2-y^2}$ order parameter, and the likewise real quantity Δ^1 denotes the induced d_{xy} component. Hereafter we assume that Δ^0 is a robust order parameter that develops true long-range order at $T < T_c \simeq 90$ K and remains ordered in all of the phases we discuss. The impurity spin S_i^z uniquely determines the relative phase of the induced order parameter to be $+\pi/2$ for $S^z = +1$ and $-\pi/2$ for $S^z = -1$. The main result of the previous section was that the impurity spins, considered independently of the coupling to the induced order parameter, will exhibit a spin glass phase at low temperatures. Let us next consider what effects the inter-patch Josephson interaction may have.

In order to address this question we examine the Josephson coupling between different patches given by

$$H^J = - \sum_{ij} |I_{ij}| (\Psi_i^* \Psi_j + \Psi_i \Psi_j^*), \quad (8)$$

Using the local order parameter Eq. (7) this leads to an interaction of the form

$$H^J = -2 \sum_{ij} |I_{ij}| [(\Delta_i^0)^2 + (\Delta^1)^2 S_i^z S_j^z] \cos(\theta_i - \theta_j) \quad (9)$$

We have here assumed that $\Delta_i^0 = \Delta_j^0 = \Delta^0$, and likewise for Δ_1 . The first term, which is zeroth order in Δ_1 , wants to align the phase of Δ_i^0 at different patches by setting $\theta_i = \theta_j$. The second term wants to align the impurity spin in an ferromagnetic phase. The Josephson coupling thus favors a ferromagnetic spin configuration, while the effective RKKY and dipolar spin model favors

a spin glass. In the next section we will consider some possible outcomes of this competition.

We would, however, like to point out that the RKKY and Josephson effects are not as independent as they may at first seem. Both are mediated by electrons and while the dominant part of the RKKY interaction in the superconducting state is zeroth order in Δ^1 there is a part that is second order in Δ^1 , corresponding to the Josephson coupling. The Josephson interaction physically expresses an effect arising from electron pair tunneling, while the GG part of the RKKY interaction expresses an electron-hole channel. Here G and F are the normal and anomalous propagators in superconducting state. The FF part of the RKKY interaction is closer to the Josephson interaction since it expresses an electron-electron process. Furthermore, there is a second order contribution to the RKKY interaction which we have not evaluated in this work. This contains two explicit d_{xy} propagators, with two exchange interactions at the impurity spins. This part of the interaction should be similar to the Josephson interaction since explicit information about the patches, such as spatial decay, should be contained in the propagators. The physical effects of this term should, however, be included in the Josephson coupling considered above.

IV. POSSIBLE PHASES

In this section we will consider the different phases that could occur as a result of the competition between the RKKY and the Josephson terms. First we will enumerate the different phases and thereafter we will discuss them in some more detail.

The simplest phase is a ferromagnetic (FM) phase, favored by the Josephson coupling of different patches. The FM phase is characterized by a finite spatial average of the magnetization and the induced phase; $\langle S^z \rangle \neq 0$ and $\langle \Delta_1 \rangle \neq 0$.

The phase favored by the effective spin Hamiltonian, on the other hand, is a SG phase, where the spatial average of the magnetization vanishes

$$\langle S^z \rangle = 0, \quad (10)$$

but the time averaged local magnetization remains finite;

$$\langle S_i^z \rangle_\tau \neq 0. \quad (11)$$

Assuming that the phase of Δ^1 is determined by the impurity spin this gives rise to similar ordering for the induced phase; $\langle \Delta^1 \rangle = 0$ and $\langle \Delta^1 \rangle_\tau \neq 0$. In this case the combination $S^z \Delta^1$ will also have a non-vanishing spatial average

$$\langle S^z \Delta^1 \rangle \neq 0 \quad (12)$$

This particular order parameter is unique for the coupling of SG and SC degrees of freedom and is not present in a purely magnetic system. It describes the phase-locking of

the superconducting phase and the impurity spin.⁹ There also exists the possibility that the induced order parameter Δ^1 does not phase-lock with the impurity spin, but prefers to vary over length scales greater than the inter-patch distance.¹⁰ In that case the spatial average $\langle S^z \Delta^1 \rangle$ would vanish, but the local time average $\langle S_i^z \Delta_i^1 \rangle_\tau$ would still remain finite.

If quantum fluctuations are strong enough, then it would also be possible to realize a paramagnetic spin phase. In this case also the time averages $\langle S^z \rangle_\tau$ and $\langle \Delta^1 \rangle_\tau$ would vanish, and only local $\langle S^z \Delta^1 \rangle$ would remain finite, for as long as the phase-locking is maintained. In this work we will, however, neglect this possibility since we focus on large classical spins.

After having considered the different options let us consider the interplay between the different terms. If there is a low concentration of impurity spins (the patches do not overlap), then the ferromagnetic Josephson term is bound to be exponentially small, and the ground state should be a spin glass, characterized by a non-vanishing spatial average $\langle S^z \Delta^1 \rangle$. If the impurity concentration becomes larger, but not large enough to kill the Δ^0 superconductivity, then there may be a region where the ferromagnetic Josephson terms are predominant and the FM phase is formed. The FM phase was considered in detail in a previous Ginzburg-Landau description⁴.

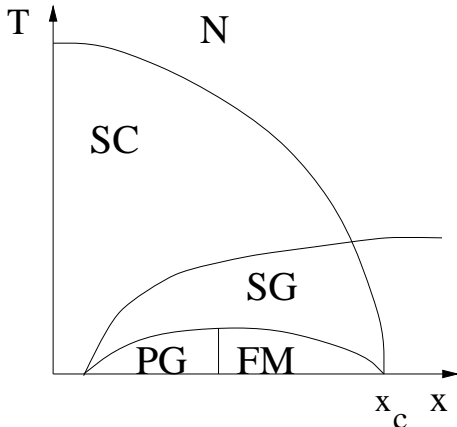


FIG. 2. Phase diagram for the impurity doped d-wave superconductor as function of impurity doping (x) and temperature (T). The labeled diagrams show the normal (N) phase, the $d_{x^2-y^2}$ superconducting (SC) phase, the spin-glass (SG) phase, the superconducting $d_{x^2-y^2} + id_{xy}$ phase-glass (PG) phase and the ferromagnetic (FM) phase. The SC phase is suppressed by disorder due to pair-breaking, while the SG phase is independent of the SC order parameter and will persist in the N phase. The PG phase is induced by the SC and SG phases and exists only within these phases. The possible FM phase is a result of strong Josephson coupling when the impurity concentration is large. It is strongly suppressed close to x_c due to the suppression of the superconducting order.

In Fig. 2 we present a phase diagram as a function of impurity concentration and temperature. Note that

this PG phase is different from previously proposed superconducting phase glasses in that the glassy behavior is only displayed in the induced component of the order parameter, and not in the robust $d_{x^2-y^2}$ part. Considering the phase diagram we note that the disorder suppresses the critical temperature for the superconductor-metal insulator, as is well known. The impurity spins form a spin-glass phase at low temperatures, and this phase is independent of the electronic order parameter and persists also in the metallic phase. The PG phase is induced by the SG and SC phases and hence it must only exist within the boundaries of these two phases. The possible FM phase is induced by a large impurity concentration. As the $d_{x^2-y^2}$ superconducting order parameter gets suppressed by disorder the induced component will also vanish, and hence the FM phase gets strongly suppressed as we approach x_c .

Experimentally the proposed phase locked state can be observed in scanning tunneling microscope measurements, where the particle spectrum would develop a full gap near impurity site even though the phase of Δ^1 remains uncertain⁴. The ac magnetic susceptibility in the superconducting state also should show features upon crossing the SG and PG lines. Another experiment, that would be sensitive to the appearance of the $\langle S^z \Delta^1 \rangle$, would be the penetration depth that would become exponential $\delta l / \lambda^2 \propto \exp[-|\Delta^1|/T]$ in the PG and FM phases.

V. CONCLUSION

We have examined the role of magnetic impurities in a d-wave superconductors. In particular we have studied the effective impurity spin model arising from electron mediated RKKY and magnetic dipolar terms and argued that these terms lead to a SG phase at low temperatures. Furthermore we have analyzed the coupling between the spin- and superconductor order parameter that arises from the Josephson interaction of patches of induced order parameter around the impurity spins. The Josephson interaction favors a FM phase. At high impurity concentrations the FM phase may be realized, while at low concentrations the SG phase would be preferred. Due to the coupling between the spin and superconducting order parameters the SG phase induces a superconducting PG at low temperatures. The glassy behavior is a property of the induced d_{xy} component of the order parameter, while the primary component $d_{x^2-y^2}$ is assumed robust. The superconducting PG phase is characterized by an order parameter of the form $\langle S^z \Delta^1 \rangle$, which describes the phase-locking of the induced order parameter and the local impurity spin. In addition to the PG and FM phases we have also discussed a possible paramagnetic phases.

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APPENDIX A: THE RKKY INTERACTION

The RKKY interaction describes a second-order process, where an electron of momentum k interacts with an impurity spin at $r = R_1$, is scattered to a state with momentum q and interacts with another impurity spin at $r = R_2$, where it is scattered back into the original state. This process is described by the following diagram:

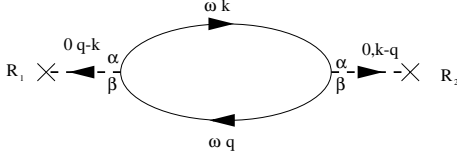


FIG. 3. Second-order RKKY interaction.

Using Feynman rules we get the following expression for the effective Hamiltonian

$$-iH = (-1) \int \frac{d^d k}{(2\pi)^d} \int \frac{d^d q}{(2\pi)^d} \int \frac{d\omega}{(2\pi)} \times iG^0(\omega, k)(-iV_{k\alpha, q\beta}^{R1})iG^0(\omega, q)(-iV_{q\beta, k\alpha}^{R2}) \quad (A1)$$

$$iH = \int \frac{d^d k}{(2\pi)^d} \int \frac{d^d q}{(2\pi)^d} \int \frac{d\omega}{(2\pi)} \times G^0(\omega, k)V_{k\alpha, q\beta}^{R1}G^0(\omega, q)V_{q\beta, k\alpha}^{R2} \quad (A2)$$

Using the Nambu formalism for the superconductor this expression transforms to

$$H = \int \frac{d^2 k}{(2\pi)^2} \int \frac{d^2 q}{(2\pi)^2} \int \frac{d\omega}{(2\pi i)} \times \text{Tr} \left\{ [G(\omega, k)][V_{k,q}^{R1} \tau_0][G(\omega, q)][V_{q,k}^{R2} \tau_0] \right\}, \quad (A3)$$

where

$$G(\omega, k) = \begin{bmatrix} G_{11}(\omega, k) & F(\omega, k) \\ F(\omega, k) & G_{22}(\omega, k) \end{bmatrix} \quad (A4)$$

$$\tau_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (A5)$$

$$F(\omega, k) = \frac{\Delta_k}{\omega^2 - E_k^2 + i\delta} \quad (A6)$$

$$G_{11}(\omega, k) = \frac{\omega + \epsilon_k}{\omega^2 - E_k^2 + i\delta} \quad (A7)$$

$$G_{22}(\omega, k) = \frac{\omega - \epsilon_k}{\omega^2 - E_k^2 + i\delta}, \quad (A8)$$

where $E_k = \sqrt{\epsilon_k^2 + \Delta_k^2}$. For a tight binding model $\epsilon_k = -t[\cos(k_x a) + \cos(k_y a)]$, and the $d_{x^2-y^2}$ gap is given by $\Delta_k = \Delta_0 [\cos(k_x a) - \cos(k_y a)]$.

Next we will consider a few specific forms of the interaction $V_{k\alpha, q\beta}^R$, which is the Fourier transform of the electron-impurity spin interaction. For free electrons $|k\rangle = e^{ikx}$ and we get

$$V_{k\alpha, q\beta}^R = \langle k\alpha | V^R | q\beta \rangle = \int d^d x e^{-i(k-q)x} \langle \alpha | V^R | \beta \rangle \quad (A9)$$

Let us first consider a contact potential of the form $V^R = J\delta(x-R)S \cdot I$, where S denotes the electron spin and I the impurity spin. This results in

$$V_{k\alpha, q\beta}^R = J e^{-i(k-q)R} \langle \alpha | S \cdot I | \beta \rangle \quad (A10)$$

Next we will consider a $L \cdot I$ interaction, where L is the angular momentum of the electron, and I is the spin of the impurity at location R . The electron moves relative to the impurity spin, which sees a magnetic field of the form $B \propto \frac{L}{|x-R|^3}$, where x is the location of the electron. We will consider two spatial dimensions, and since the angular momentum of the electron will have only a z -component we consider an interaction of the form

$$V^R = J \frac{L \cdot I}{|x-R|^3} = J \frac{L^z I^z}{|x-R|^3}, \quad (A11)$$

where $L = (x-R) \times p = -i(x-R) \times \nabla_x$. We get the following expression for the matrix element:

$$V_{k\alpha, q\beta}^R = J \int d^2 x e^{-ikx} \frac{[-i(x-R) \times \nabla_x]^z}{|x-R|^3} I^z e^{iqx} \langle \alpha | \beta \rangle \quad (A12)$$

After performing the integrals we arrive at

$$V_{k\alpha, q\beta}^R = -2\pi i J I^z e^{-i(k-q)R} \frac{(k \times q)^z}{|k-q|} \delta_{\alpha, \beta} \quad (A13)$$

We are now in a position to evaluate the effective interaction

$$H = -i \int \frac{d^2 k}{(2\pi)^2} \int \frac{d^2 q}{(2\pi)^2} \int \frac{d\omega}{(2\pi)} V_{k,q}^{R1} V_{q,k}^{R2} \times [G_{11}(\omega, k)G_{11}(\omega, q) + G_{22}(\omega, k)G_{22}(\omega, q) + 2F(\omega, k)F(\omega, q)]. \quad (A14)$$

We will begin with the frequency integral for the FF contribution

$$I_F = \int \frac{d\omega}{(2\pi)} 2F(\omega, k)F(\omega, q). \quad (A15)$$

There are poles at $\omega = \pm \sqrt{E_k^2 - i\delta} = \mp E_k \pm i\delta$. Closing the integral in the upper half plane leads to contributing poles at $\omega = -E_k + i\delta$ and $\omega = -E_q + i\delta$. It follows that

$$I_F = i \frac{\Delta_k \Delta_q}{E_k E_q (E_k + E_q)} \quad (A16)$$

Next we will consider the GG contribution

$$I_G = \int \frac{d\omega}{(2\pi)} G_{11}(\omega, k) G_{11}(\omega, q) + G_{22}(\omega, k) G_{22}(\omega, q) \quad (\text{A17})$$

Proceeding as above it follows that

$$I_G = i \frac{(\epsilon_k \epsilon_q - E_k E_q)}{E_k E_q (E_k + E_q)}. \quad (\text{A18})$$

We will start by considering an interaction of the L cot I kind. We then have

$$V_{k,q}^{R_1} V_{q,k}^{R_2} = 4\pi^2 J^2 I_1^z I_2^z e^{-i(k-q)r} \left| \frac{k \times q}{k - q} \right|^2, \quad (\text{A19})$$

where $r = R_1 - R_2$. For the effective interaction we then get

$$H = \frac{J^2 I_1^z I_2^z}{4\pi^2} \int d^2 k \int d^2 q e^{-i(k-q)r} \times \left| \frac{k \times q}{k - q} \right|^2 \frac{(\Delta_k \Delta_q + \epsilon_k \epsilon_q - E_k E_q)}{E_k E_q (E_k + E_q)} \quad (\text{A20})$$

In order to solve the above integral we introduce the nodal point approximation.¹¹ The dominant contribution to the integral should come from each nodal point, and we perform a rotation and translation to transform the origin to the nodal points, with the x -axis along the tight binding Fermi surface, see Fig. 4.

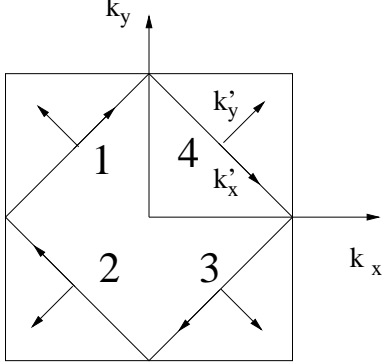


FIG. 4. Node notation.

The transformation is given by

$$\begin{pmatrix} k'_x \\ k'_y \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} k_x \\ k_y \end{pmatrix} - \begin{pmatrix} 0 \\ k_F \end{pmatrix}, \quad (\text{A21})$$

where $k_F = \pi/\sqrt{2}a$ and $\theta = \{\pi/4, 3\pi/4, 5\pi/4, 7\pi/4\}$ for the four nodes.

Next we need to apply these transformations to all quantities in the effective interaction. The energy of a tight-binding model, $\epsilon_k = -t [\cos(k_x a) + \cos(k_y a)]$, will transform according to $\epsilon_k = v_F k'_y$ for all nodes, and the gap function, $\Delta_k = \Delta_0 [\cos(k_x a) - \cos(k_y a)]$ will transform according to $\Delta_k = \pm v_\Delta k'_x$, with a positive sign for

nodes 1 and 3, and a negative sign for nodes 2 and 4. Assuming an isotropic superconductor $v = v_F = v_\Delta$ this leads to $E_k = vk'$.

The rotation and translation cannot affect $|k - q|$, and therefore $|k - q| = |k' - q'|$. Furthermore

$$\begin{aligned} (k \times q)^z &= k_x q_y - k_y q_x \\ &= k'_x q'_y - k'_y q'_x + k_F (k'_x - q'_x) \end{aligned} \quad (\text{A22})$$

for all the four nodes. The second term is linear in the momentum and will be retained. The final term to be transformed, $(k - q)r$, is considered next. This term depends on the direction of r , and different nodes will give different contributions. Assume that r is fixed in a direction θ_r , and the contribution to the effective interaction from node 1 has been calculated. Since all other terms are identical, the contribution from the other nodes must be given by substituting $\{\theta_r + \pi/2, \theta_r + 2\pi/2, \theta_r + 3\pi/2\}$ for θ_r in the expression obtained for node 1. Therefore we will look at how the transformation is done for node 1, and the results for the other nodes will follow. For node 1 we get

$$\begin{aligned} (k - q)r &= r_x (k_x - q_x) + r_y (k_y - q_y) \\ &= \frac{r|k' - q'|}{\sqrt{2}} [\cos \theta_{k' - q'} (\cos \theta_r + \sin \theta_r) \\ &\quad + \sin \theta_{k' - q'} (-\cos \theta_r + \sin \theta_r)] \end{aligned} \quad (\text{A23})$$

As a summary we have thus arrived at the following transformations

$$\begin{aligned} \epsilon_k &= vk'_y \\ \Delta_k &= vk'_x \\ E_k &= vk' \\ |k - q| &= |k' - q'| \\ (k \times q)^z &= k_F (k'_x - q'_x) \\ (k - q)r &= \frac{r|k' - q'|}{\sqrt{2}} [\cos \theta_{k' - q'} (\cos \theta_r + \sin \theta_r) \\ &\quad + \sin \theta_{k' - q'} (-\cos \theta_r + \sin \theta_r)] \end{aligned}$$

Dropping the primes, and using these results we can thus linearize the effective interaction around the nodes:

$$H = \frac{J^2 I_1^z I_2^z}{4\pi^2} \int d^2 k \int d^2 q e^{-i(k-q)r} \times \frac{k_F^2 |k - q|^2 \cos^2 \theta_{k-q} v k_x v q_x + v k_y v q_y - v k v q}{|k - q|^2 v k v q (v k + v q)} \quad (\text{A24})$$

We start by integrating over the angles. First we fix the relative angle θ between k and q and integrate over θ_k , thereafter we integrate over θ . Integrating over θ_k we get

$$H = \frac{J^2 I_1^z I_2^z k_F^2}{4\pi^2 v} \int dk \int dq \frac{kq}{k + q} \int d\theta (\cos \theta - 1) \times \pi [J_0(r|k - q|) - \sin(2\theta_r) J_2(r|k - q|)] \quad (\text{A25})$$

The angular dependence will, however, vanish, since summing up the contributions from the four nodes gives us

$$\sin(2\theta_r) + \sin(2(\theta_r + \pi/2))$$

$$+ \sin(2(\theta_r + \pi)) + \sin(2(\theta_r + 3\pi/2))] = 0, \quad (\text{A26})$$

and this tells us that the effective interaction is isotropic, even though the gap is anisotropic. Integrating out the relative angle we find

$$H = \frac{2J^2 I_1^z I_2^z k_F^2}{v} \int dk \int dq \times \frac{kq}{k+q} [-J_0(kr)J_0(qr) + J_1(kr)J_1(qr)] \quad (\text{A27})$$

This integral is oscillatory, and we introduce a momentum cut-off K and make the integration variables dimensionless by letting $k \rightarrow rk$ and $q \rightarrow rq$:

$$H = \frac{2J^2 I_1^z I_2^z k_F^2}{vr^3} \int_0^{Kr} dk \int_0^{Kr} dq \quad (\text{A28})$$

$$\frac{kq}{k+q} [-J_0(k)J_0(q) + J_1(k)J_1(q)], \quad (\text{A29})$$

This integral can be solved numerically, and the behavior for large r is given by

$$H \sim \frac{2J^2 I_1^z I_2^z k_F^2}{vr^3} [0.37 + 0.33 \sin(2Kr)]. \quad (\text{A30})$$

This results represents an anti-ferromagnetic r^{-3} part that is independent of the cut-off, and an oscillating $\sin(2Kr)r^{-3}$ part. Due to the relative sizes of the two terms the interaction is always positive and hence completely anti-ferromagnetic. The $S \cdot I$ interaction will lead to a very similar result, differing only in the prefactor. This can be seen, since the result, after performing the first angular integral will be given by

$$H = \frac{J^2 I_1 \cdot I_2}{4\pi^3 v} \int dk \int dq \frac{kq}{k+q} \times \int d\theta (\cos \theta - 1) \pi J_0(r|k - q|) \quad (\text{A31})$$

and the only difference compared to the $L \cdot I$ interaction is the prefactor. The final result for the $S \cdot I$ coupling will be

$$H^{S \cdot I} \sim \frac{J^2 I_1 \cdot I_2}{2\pi vr^3} [0.37 + 0.33 \sin(2Kr)] \quad (\text{A32})$$

So the two electron-spin couplings lead to the same functional form of the RKKY interaction. We have therefore showed that both the $S \cdot I$ and $L \cdot I$ interactions give rise to an effective anti-ferromagnetic model. We have used the nodal approximation and assumed that the superconductor is isotropic $v_F = v_\Delta$, and these approximations may change the final result somewhat, but it appears unlikely that they would make the effective spin-spin interactions predominantly ferromagnetic, and therefore the randomly distributed spins will form a spin-glass phase, as discussed in the main part of the paper.

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